Enrollment No: _____ Exam Seat No: _____

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name: Numerical Analysis

Subject Code: 4SC03NUA1 **Branch: B.Sc. (Mathematics, Physics)**

Semester: 3 Date: 04/12/2018 **Time:02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) $\Delta^2 y_0 =$_____. (01)

(1) $\Delta y_0 - \Delta y_1$ (2) $y_2 - 2y_1 + y_0$
(3) $y_2 + 2y_1 + y_0$ (4) $\Delta y_2 - \Delta y_0$

b) $\nabla =$ _____. (01)

(1) $1 + \mathrm{E}$ (2) $1 - E$
(3) $1 + E^{-1}$ (4) $1 - E^{-1}$

c) Averaging operator is denoted by _____. (01)

(1) S (2) δ
(3) E (4) μ

d) $[x_1, x_2] =$ _____. (01)

(1) $\dfrac{x_2 - x_1}{y_2 - y_1}$ (2) $\dfrac{y_2 - y_0}{x_2 - x_0}$
(3) $[x_2, x_1]$ (4) None of these

e) $[x_0, x_1, x_2, x_3] =$ _____. (01)

(1) $\dfrac{3!}{h^3}\Delta^3 y_0$ (2) $\dfrac{h^3}{3!}\nabla^3 y_3$
(3) $\dfrac{1}{3!\,h^3}\nabla^3 y_3$ (4) $\dfrac{1}{3!\,h^3}\Delta^3 y_0$

f) $\delta =$ _____. (01)

(1) $\mathrm{E}^{\frac{1}{2}} - E^{-\frac{1}{2}}$ (2) $\mathrm{E}^{\frac{1}{2}} + E^{-\frac{1}{2}}$
(3) $\Delta E^{-\frac{1}{2}}$ (4) $\Delta E^{\frac{1}{2}}$

g) Which relation is correct (01)

(1) $\Delta^3 y_1 = \nabla^2 y_2$ (2) $\Delta^3 y_1 = \nabla^3 y_2$
(3) $\Delta^3 y_2 = \nabla^4 y_5$ (4) $\Delta^3 y_2 = \nabla^3 y_5$

h) If $\Delta y_5 = 5$ and $y_6 = 11$, then $y_5 =$ _____. (01)

(1) 16 (2) 6

(3) -16 (4) -6

i) Bessel's formula is most appropriate when p lies between_____. (01)

(1) -0.25 and 0.25 (2) 0.75 and 1.00

(3) 0.25 and 0.75 (4) None of these

j) If $y = 3x^3 - 2x^2 + 1$, then $\Delta^3 y =$ _____. (01)

(1) 9 (2) 18

(3) -4 (4) None of these

k) $E = e^{-hD}$. Determine whether the statement is True or False. (01)

l) Stirling's interpolation formula is used to estimate the value of a function near the middle of a table. Determine whether the statement is True or False. (01)

m) The n^{th} divided differences of a polynomial of degree n is constant. Determine whether the statement is True or False. (01)

n) The number 1.5034 is correct up to four significant digits. Determine whether the statement is True or False. (01)

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) State and prove Newton's backward interpolation formula. (07)

b) Express $f(x) = 3x^4 - 16x^3 + 22x^2 - 29x + 5$ in factorial notation. (04)

c) If $\frac{5}{6}$ be represented approximately by 0.8333, then find relative error and percentage error. (03)

Q-3 **Attempt all questions** **(14)**

a) State and prove Gauss's forward interpolation formula. (07)

b) Given the following data (04)

x	-1	0	2	5
$f(x)$	9	5	3	15

Find $f(x)$ assuming it to be a polynomial of degree two in x.

c) Round-off the following numbers correct up to 4 significant figures: (03)
(1) 0.34026, (2) 4.501, (3) 2456.782, (4) 8.0008, (5) 0.0055672, (6) 40.3585.

Q-4 **Attempt all questions** **(14)**

a) State and prove Everett's interpolation formula. (07)

b) Determine the interpolating polynomial of degree three using Lagrange's interpolation for the table below : (04)

x	-1	0	1	3
y	2	1	0	-1

c) If $y_0 = 3, y_1 = 12, y_2 = 81, y_3 = 2000$ and $y_4 = 100$, then determine $\Delta^4 y_0$. (03)

Q-5 **Attempt all questions** **(14)**

a) State and prove Lagrange's interpolation formula. (05)

b) If $u = \frac{4x^2y^3}{z^4}$ and errors in x, y, z be 0.001, then compute the relative maximum error in u when $x = y = z = 1$. (05)

c) Given the following table (04)

x	2	3	4	5
$f(x)$	14.5	16.3	17.5	18.0

Construct the difference table and compute $f(2.5)$ by Newton's forward interpolation formula.

Q-6 **Attempt all questions** **(14)**

a) State and prove Newton's divided-differences interpolation formula. (05)

b) Use Stirling's formula, to compute $f(0.345)$ from the following data: (05)

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	2.74560	2.82922	2.97427	3.18993	3.49034

c) If value of x and $\log_{10} x$ are $(300, 2.4771), (304, 2.4829), (305, 2.4843)$ and $(370, 2.4871)$. Find the value of $\log_{10} 301$ by Newton's divided-differences interpolation formula. (04)

Q-7 **Attempt all questions** **(14)**

a) Prove that $\Delta^n [x]^n = n!\, h^n$ and $\Delta^{n+1}[x]^n = 0$. (05)

b) Prove that (05)

(1) $\left(E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right)(1+\Delta)^{\frac{1}{2}} = 2 + \Delta$,

(2) $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{\delta^2}{4}}$.

c) The population of a town is given below. Apply Gauss'sbackward interpolation formula to get population in 1926. (04)

x	1911	1921	1931	1941	1951
y	15	20	27	39	52

Q-8 **Attempt all questions** **(14)**

a) Prove that (05)

(1) $\Delta[f(x) \cdot g(x)] = g(x+h) \cdot \Delta f(x) + f(x) \cdot \Delta g(x)$,

(2) $\Delta\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\cdot\Delta f(x) - f(x)\cdot\Delta g(x)}{g(x+h)\cdot g(x)}$ $[g(x) \neq 0]$.

b) Given the following table (05)

x	20	30	40	50
y	512	439	346	243

Find the value of $y(35)$ by Bessel's formula.

c) Using inverse interpolation find the value of x for $y = 5$ from the following table (04)

x	1	3	4
y	3	12	19